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1995 J. Phys. A: Math. Gen. 28 3005

(http://iopscience.iop.org/0305-4470/28/11/006)

J. Phys. A: Math. Gen. 28 (1995) 3005–3013. Printed in the UK

Suppression of phase separation in quenched turbulent binary liquids

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Received 21 October 1994

Abstract. Giving importance to the *active* participation of concentration fluctuations in the dynamics, a phenomenological model is proposed to describe the experimentally observed suppression of phase separation when a turbulent binary fluid mixture is quenched below its consolute point T_c. It is assumed that an inverse cascade of concentration fluctuations is acted upon by the tubulence-generated viscous shear together with a loss due to the presence of Alfvén-like waves, which cuts off the inverse cascade at some scale. As a consequence, The apparent depression of critical temparature follows the power law $\Delta T_c \sim R^{\lambda}$ with $\lambda = 1.44$ and 2.4 in two extreme theoretical limits (R = Reynolds number). This result is comparable to the experimental values of 1.4 and 2.1 in two different measurements.

1. Introduction

1.1. The experiment

A few years ago, Pine *et al* [14] observed that when a turbulent (randomly stirred) binary fluid mixture (3-methylpentane + nitroethane) is suddenly quenched below its consolute point ($T_c = 26.4\,^{\circ}\mathrm{C}$), the phase separation (which is usually observed in an unstirred mixture) is tremendously inhibited, showing no attenuation (and hence no scattering from any concentration fluctuations) of the laser beam (632.8 nm wavelength) transmitted through the turbulent mixture, until the quenching temperature is low enough for the phase separation to show up through an attenuation of the transmitted laser beam. They identified the Reynolds number (R) dependent temperature, $T_c'(R)$, at which the transmitted intensity fell by 15%, as the new (depressed) critical temperature of the turbulent system. Further, they observed that the depression $\Delta T_c = T_c - T_c'(R)$ obeys the power law

$$\Delta T_c \sim R^{\lambda} \tag{1}$$

where λ, presumably a universal number, was found to be equal to 2.1 and 1.4 in two different measurements.

1.2. Expected physical processes

The quenched randomly stirred mixture involves two simultaneous physical processes. First, since the mixture in the experiment is violently stirred at large scales, we expect the existence of the universal Kolmogorov (direct) self-similar cascade of energy [6, 12, ch 21] from larger to smaller (hydrodynamic) scales. Dimensional arguments suggest the corresponding spectrum to be

$$E(k) \sim \bar{\varepsilon}^{2/3} k^{-5/3} \tag{2}$$

0305-4470/95/113005+09$19.50

the inverse timescale associated with this cascade being

$$\nu(k)k^2 \sim \bar{\varepsilon}^{1/3} k^{2/3} \tag{3}$$

where $\bar{\varepsilon}$ is the energy injection-rate at large scales ($\sim L$, say). This cascade is terminated at the Kolmogorov wavenumber $k_d = 1/\eta$, given by

$$\eta = \left(\frac{\nu_0^3}{\bar{\varepsilon}}\right)^{1/4} = \frac{L}{R^{3/4}} \tag{4}$$

due to dissipation by viscosity, where ν_0 is the kinematic viscosity of the fluid, and R is the macroscopic Reynolds number. In the experiment, $L = 2.5$ cm and R was varied in the range 6.0×10^3–4.5×10^4.

The other physical process is the spinodal decomposition, which takes place at very small (thermodynamic) scales (where, as we shall see, the effect of the stirring can be neglected, the microscopic Reynolds number being too small at these scales), resulting from the Cahn–Hilliard instability [3,4] as a result of the quenching below T_c. Concentration fluctuations grow in size in the initially homogeneous mixture, with maximum growth rate occurring at a wavenumber k_m. The growth rate $\Gamma(k)$ is related to the diffusivity D_T as

$$\Gamma(k) \approx D_T k^2 \tag{5}$$

with

$$D_T \approx k_B T/6\pi\eta_0\xi(T) \tag{6}$$

where η_0 is the molecular viscosity and the corrrelation length $\xi(T)$ is given by

$$\xi(T) = \xi_0 \left|\frac{T_c}{T_c - T}\right|^{\bar{\nu}}. \tag{7}$$

Since one can assume [11] $k_m \approx 1/\xi(T)$, taking $\bar{\nu} = \frac{5}{8}$, $\xi_0 = 2.28$ Å, $T_c = 26.4\,^{\circ}$C, and $T_c - T \approx 20$ mK [14], equation (7) leads to $k_m \approx 10^7$ m^{-1}.

In section 2 we outline the various theories that attempted to find out the exponent λ of equation (1), corresponding assumptions, and final results. In section 3 we set out our main observations, consequent assumptions, and the proposed phenomenological calculations.

2. Previous theories

A rough estimate of the exponent λ from the above data has already been made by Pine *et al* [14]. It involves in assuming that the phase separation is inhibited when the timescale determined by the *typical* viscous strain rate $S(k_d) = \nu_0 k_d^2$ matches the *maximum* growth rate $\Gamma(k_m) = D_T k_m^2$ at $T = T_c'(R)$, i.e. $D_T k_m^2 \approx \nu_0 k_d^2$. Using (6), (4), and $k_m \approx 1/\xi$, one obtains $R^{3/2} \sim 1/\xi^3(T_c')$, which upon using (7) leads to $T_c - T_c'(R) \sim R^{1/2\bar{\nu}}$. Taking $\bar{\nu} = \frac{5}{8}$ gives the value $\lambda = 1/2\bar{\nu} = 0.8$. This value is much less than the experimental value (~ 2).

Onuki [13] goes a little further in estimating λ by assuming that the turbulent velocity field in the dissipation range $k > k_d$ can be expanded [2] as a Taylor expansion in space, and assuming the coefficient of the linear term behaving like a white noise [9, 10]. His theory, however, introduces no major improvement over the above value, yielding $\lambda = 0.9$–1.0.

Aronovitz and Nelson [1] also have put forward a model of the above suppression of phase separation, assuming the mixture to behave like a *passive* mixture. By carrying out a linear stability analysis, they found a depression of the critical temperature T_c in the presence of stirring.

Satten and Ronis [16] also assumes the mixture to be *passive*. Neglecting the non-linearities (associated with both the self-advection of the velocity field, i.e. $(u \cdot \nabla)u$, and *active* participation of concentration gradients, i.e. $\nabla\phi\,\nabla^2\phi$) in the dynamics of the velocity field (cf equation (8)), they carried out a renormalization group (RG) calculation leading to a new fixed point, and a value $\lambda = 1.74$. This value is in close agreement with the experimental value (~ 2).

The assumption of a *passive* dynamics in the above RG approach is, however, quite contrary to what Ruiz and Nelson [15] had already argued, namely, the *active* participation of concentration fluctuations *cannot* be ignored at small scales. It is to be noted that the relevant physical processes, under the experimental conditions, are confined to the *large* wavenumber range $k_d < k < k_m$. Therefore, in the dynamics of the velocity field, although the advective non-linearity $(u \cdot \nabla)u$ is not important in this (viscous) range, the term due to active participation, $\nabla\phi\,\nabla^2\phi$), having three space derivatives, does become important in this *small-scale* regime. Further, the RG formulation had the difficulty that the Reynolds number in the scaling relation $\Delta T_c \sim R^\lambda$ (equation (1)) is not directly obtainable.

3. Proposed model

Here we show that a simple phenomenological model, based on the pictures provided by Ruiz and Nelson [15], can be used to calculate the value of the exponent λ in (1), in reasonable agreement with the experiment. We set out our observations and consequent assumptions in subsection 3.1, while the calculations will be presented in later subsections.

3.1. Observations and assumptions

From the calculations following (4) and (7), we see that there is enough room to allow for many physical processes between the wavenumbers $k_d \sim 10^5\ \mathrm{m}^{-1}$ and $k_m \sim 10^7\ \mathrm{m}^{-1}$. Further, the wavelength of the laser light used in the experiment was $\Lambda = 632.8$ nm, giving a probing wavenumber $K = \Lambda^{-1} \sim 10^6\ \mathrm{m}^{-1}$, which lies between k_d and k_m. At a quenching temperature $T = T_c'(R)$, negligible attenuation (by 15%) of the transmitted laser beam was observed. Thus, as suggested by Pine *et al* [14], we assume that any phase separation is confined to the wavenumbers larger than K. Since $K > k_d$, the phase separation is confined *only* in the dissipation range ($k > k_d$).

Further, the microscopic Reynolds number at the dissipation wavenumber should be $R(k_d) \approx 1$ (by definition), and $k_m \approx 10^2 k_d$ (in the experiment). Since $R(k) \sim k^{-1}$, this leads to a *very small* value of the microscopic Reynolds number at the spinodal wavenumber, namely $R(k_m) \approx 10^{-2}$. Therefore, one can safely neglect any effects of the stirring in the dynamics of spinodal decomposition, which takes place at and above wavenumbers $\sim k_m$. Consequently, we assume that the near-equilibrium dynamics of spinodal decomposition [5] is still valid at and above wavenumbers $\sim k_m$, i.e. in the mechanism responsible for production of concentration fluctuations.

In addition, we take a simple picture of the physical processes leading to the suppression of phase separation in $k < k_m$ based on the ideas put forward by Ruiz and Nelson [15]. The spinodal decomposition injects, as Ruiz and Nelson had already suggested, concentration fluctuations at wavenumbers k_m (at the maximum rate). This is an ideal condition [15] to support an *inverse* cascade of concentration fluctuations in $k < k_m$. This cascade is possible *only* in an *active* binary liquid [15]. Since at $T = T_c'(R)$, as indicated earlier, the phase separation takes place only above the laser wavenumber K, we shall assume that the cascade is stopped at K by the turbulence-generated viscous shear. Dissipation of the concentration

fluctuations takes place through conversion (due to the presence of Alfvén-like waves) of concentration gradients into velocity fluctuations and its subsequent dissipation by viscosity. This dissipation, together with the production at k_m, maintains a steady state.

Further, this phenomenological model does not have any difficulty in finding the required dependence on the Reynolds number R, because it in fact makes use of (4) to find the required dependence.

3.2. Active dynamics and phenomenology

The dynamics of a symmetric binary fluid, as Ruiz and Nelson [15, section VI] suggest, is governed by the following equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \lambda_0 (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\frac{\nabla P'}{\rho_0} - \alpha_0 \nabla\phi \nabla^2\phi + \nu_0 \nabla^2 \boldsymbol{u} + \boldsymbol{f} \tag{8}$$

$$\frac{\partial \phi}{\partial t} + \tilde{\lambda}_0 (\boldsymbol{u} \cdot \nabla)\phi = \Gamma_0 \nabla^2 \frac{\delta F\{\phi\}}{\delta \phi} + \zeta \tag{9}$$

along with the condition for incompressibility $\nabla \cdot \boldsymbol{u} = 0$, and

$$F\{\phi\} = \int \mathrm{d}^3x \ \left\{\tfrac{1}{2} |\nabla\phi|^2 + \tfrac{1}{2} r \phi^2 + \tfrac{1}{4!} g \phi^4\right\} \tag{10}$$

where Γ_0 is a positive transport coefficient. $\boldsymbol{f}(\boldsymbol{x}, t)$ and $\zeta(\boldsymbol{x}, t)$ are random stirring fields acting on the velocity field $\boldsymbol{u}(\boldsymbol{x}, t)$ and the scalar field $\phi(\boldsymbol{x}, t) = [\rho_A(\boldsymbol{x}, t) - \rho_B(\boldsymbol{x}, t)]/\rho_0$; ρ_0 is the mean density of the binary fluid composed of the two constituents A and B. The extra term in the Navier–Stokes equation signifies an *active* participation [15] of concentration gradients in the dynamics of the velocity field. Several terms are included in the effective pressure P'.

In the case of two-dimensional pure hydrodynamics, Kraichnan [8] argued about the direction of the cascades by looking at the absolute equilibrium ensemble associated with the two inviscid conservation laws; namely the total energy and the total enstrophy. Similar arguments can be applied to the present case, as Ruiz and Nelson discuss, since the symmetric binary fluid involves two conserved quantities in absolute equilibrium [15], for zero transport coefficients, and in the absence of any external stirring:

$$\begin{aligned} \hat{H} &= \frac{1}{2} \int \left\{|u(x)|^2 + \alpha_0 |\nabla\phi(x)|^2\right\} \mathrm{d}^3x = \frac{1}{2} \sum_k \left\{|u_k|^2 + \alpha_0 k^2 \phi_k^2\right\} \\ \hat{\Omega} &= \frac{1}{2} \int \phi^2(x) \mathrm{d}^3x = \frac{1}{2} \sum_k \phi_k^2 \end{aligned} \tag{11}$$

The canonical probability distribution is now given by

$$P \sim \exp\left(-\beta \hat{H} - \mu \hat{\Omega}\right) = \exp\left[-\frac{1}{2} \sum_k \left\{\beta |u_k|^2 + (\mu + \beta\alpha_0 k^2)\phi_k^2\right\}\right].$$

Equipartition among the modes requires

$$\tfrac{1}{2} \langle |u_k|^2 \rangle = \frac{1}{\beta}$$

and

$$\tfrac{1}{2} \langle \phi_k^2 \rangle = \frac{1}{\mu + \beta\alpha_0 k^2}.$$

Thus, just as in the three-dimensional pure hydrodynamic case, the energy cascade is expected to be direct. The second equipartition relation, on the other hand, suggests that there is a possibility of inverse cascade of the scalar-variance [15] corresponding to the possibility of negative 'temperature' solutions; the interpretation goes exactly as that in the absolute equilibrium of two-dimensional pure hydrodynamics.

Ruiz and Nelson also discuss about the instability associated with a phase-separating ($T < T_c$) binary mixture [15, section VI]. Since $r \propto (T - T_c)$, upon quenching the turbulent homogeneous mixture below T_c, the diffusion constant $D_0 = \Gamma_0 r$ becomes negative, and Cahn–Hilliard instability sets in [3, 4]. This instabillity starts to produce small-scale inhomogeneities (droplets) in the mixture, with a maximum growth rate occurring at a wavenumber k_m. As pointed out in subsection 3.1, since the microscale Reynolds number at k_m is only $\sim 10^{-2}$, and therefore we can safely neglect any effects of the random stirring in this instability, indicating that the near-equilibrium dynamics [5] of spinodal decomposition is still valid in this regime ($k > k_m$).

Further, Ruiz and Nelson argued that the above instability injects concentration fluctuations at scales $\sim k_m^{-1}$, which is an ideal condition for an inverse cascade larger scales (i.e. at $k < k_m$) [15, section VI]. Simple dimensional arguments yield the corresponding spectrum easily. Taking α_0 to be dimensionless makes $[u] = [\nabla\phi]$. Since

$$\langle \phi^2(x, t) \rangle = \int_0^\infty C(k)\, \mathrm{d}k \tag{12}$$

we have $[C(k)] = [L^5 T^{-2}]$ and $[\bar{\chi}] = [L^4 T^{-3}]$, where $\bar{\chi}$ is the injection rate of scalar-variance at *small* scales (*large* wavenumbers). Quite like the Kolmogorov phenomenology [6], we assume that $C(k)$ may depend only on $\bar{\chi}$ and k. Thus writing $C(k) \sim \bar{\chi}^\alpha k^\beta$, we get $4\alpha - \beta = 5$ and $3\alpha = 2$. This yields

$$C(k) \sim \bar{\chi}^{2/3} k^{-7/3}. \tag{13}$$

The inverse timescale

$$D(k)k^2 \sim \bar{\chi}^{1/3} k^{4/3}. \tag{14}$$

can also be obtained by similar dimensional arguments. (Ruiz and Nelson's EDQNM closure supports the above scaling relations, in that they give a k-independent flux for the cascade.)

3.3. Results

The phenomena discussed above suggest that there must be a crossover from the spinodal (thermodynamic) to the inverse cascade (hydrodynamic) regime at some scale. It is reasonable to take this scale as k_m^{-1}, at which the concentration fluctuations are injected. At this scale, the growth rate must match the turnover rate, i.e.

$$\Gamma(k_m) \approx D(k_m)\, k_m^2. \tag{15}$$

Using equations (5) and (14), this yields $\bar{\chi} \sim D_T^3 k_m^2$, which, using (6) and taking $k_m \approx 1/\xi(T)$, finally yields the injection rate as

$$\bar{\chi}(T) \sim (T_c - T)^{5\bar{\nu}}. \tag{16}$$

where the last step follows from (7).

3.3.1. Non-dissipative picture. As noted earlier, all the processes in the experiment (due to phase separation) are confined in the dissipation regime ($k > k_d$). Therefore, the inverse cascade is acted upon by the turbulence-generated viscous shear, with a typical strain rate $S(k_d) \simeq \nu_0 k_d^2$. Thus this cascade will be stopped when the inverse timescale of the backward cascade matches the strain rate, i.e.

$$D(k)\,k^2 \approx \nu_0 k_d^2 \tag{17}$$

which, using (14), leads to

$$[\bar{\chi}(T)]^{1/3}\,k^{4/3} \approx \nu_0\,k_d^2. \tag{18}$$

In the experiment, a new (Reynolds-number-dependent) depressed critical temperature $T_c'(R)$ is defined at which phase separation is *observed* to begin (through a 15% attenuation of the transmitted laser beam). Therefore, assuming that the inverse cascade reaches only down to the laser wavenumber K, we must have

$$[\bar{\chi}(T_c')]^{1/3}\,K^{4/3} \approx \nu_0 k_d^2. \tag{19}$$

Using (16) and (4), equation (19) leads to

$$\Delta T_c = T_c - T_c'(R) \sim R^{9/10\bar{\nu}} \tag{20}$$

giving a value

$$\lambda = 9/10\bar{\nu} = 1.44 \tag{21}$$

for $\bar{\nu} = \frac{5}{8}$.

In the above picture, the concentration fluctuations are produced at the wavenumber k_m, which then cascade down to the wavenumber K where the viscous strain stops the cascade. Thus there is no dissipation mechanism acting on the cascade and hence no steady state will be reached. Instead, concentration fluctuations will grow in the band $K < k < k_m$. In this sense, the above calculations have been performed in an extreme theoretical limit.

3.3.2. Steady-state picture. In order to maintain a steady state, the concentration fluctuations must be disssipated at the rate at which they are produced at k_m. Ruiz and Nelson's arguments [15] suggest that an indirect mechanism of dissipation can become operative in the above inverse cascade as discussed below.

Supposing the inverse cascade reaches a wavenumber K', the buildup of concentration gradient $B(k) = \sqrt{\langle|\nabla\phi|^2\rangle}$ at a wavenumber k, due to the presence of fluctuations from K' to k, is given by

$$B^2(k) = \int_{K'}^{k} q^2 C(q)\,\mathrm{d}q. \tag{22}$$

Using (13), we observe that the integral in (22) is dominated by the upper limit, and assuming $K' \ll k$, we obtain $B(k) \sim \bar{\chi}^{1/3}\,k^{1/3}$.

Now, this concentration gradient is capable of supporting Alfvén-like waves at k, with frequency $\omega(k) = B(k)\,k \sim \bar{\chi}^{1/3}\,k^{4/3}$. Since this frequency scales the same way as the inverse timescale $D(k)\,k^2$ (cf equation (14)), wave effects are important at every step of the inverse cascade.

This wave effect brings the concentration gradient modes and the velocity modes into energy-equipartition [7, 15]. (Ruiz and Nelson's simulation in fact indicated the existence of the equipartition due to the Alfvén waves in the inverse cascade regime of a miscible binary fluid on the basis of an EDQNM closure, cf [15, figure 14].) Therefore, at every step of the cascade, some amount of concentration gradient is converted into velocity mode, which

is then dissipated by the action of viscosity, because all the above processes are confined in the dissipation regime (in the experiment).

The rate of conversion of concentration gradients into kinetic energy at a wavenumber k is given by [15]

$$\omega_-(k) = \frac{B^2(k)}{\nu_0} \tag{23}$$

which, using (22) and (13), yields $\omega_-(k) \sim \bar{\chi}^{2/3} k^{2/3}/\nu_0$.

This indirect mechanism of dissipation must show up as an attenuation of the flux $\bar{\chi}$ at each step, as the cascade proceeds downward in wavenumber. At every step, $\bar{\chi}$ is attenuated by a factor $(1 - f)$, where $f = \omega_-(k)/\nu_0 k^2$. However, a fractal model involving such step-by-step dissipation, coupled with the steady-state condition, gives rise to complicated equations from which it is practically impossible to extract any information. The complication arises from the fact that $\omega_-(k)$ is determined by $B(k)$ which, in turn, is determined by $C(k)$.

However, we take a simplified picture of the steady state, in which the above dissipation mechanism is assumed to be important only in a range $k^* < k < k_m$ and no dissipation in $K < k < k^*$. The production rate $\bar{\chi}$ must match the conversion rate, i.e.

$$\bar{\chi} \approx \int_{k^*}^{k_m} \omega_-(k) C(k)\, dk \tag{24}$$

which, for the scaling $C(k) \sim k^{-7/3}$, is dominated by the *lower* limit. This suggests that we replace $\bar{\chi}$ by $\overline{\chi^*}$ when we use (13) in (24), where $\overline{\chi^*}$ is the attenuated value of the flux at k^*. Using (23), (22) and (13) with $\bar{\chi}$ replaced by $\overline{\chi^*}$, we get

$$\bar{\chi} \sim \frac{1}{\nu_0} \overline{\chi^*}^{4/3} k^{*-2/3}. \tag{25}$$

This simplified picture is equivalent to assuming that the conversion, and hence the dissipation, is *concentrated* at the wavenumber k^* because of the dominance of the lower limit. In this sense, this simplification is another extreme theoretical limit. Further, to maintain the steady state, the conversion rate $\omega_-(k^*)$ must be equal to that with the dissipation, i.e.

$$\omega_-(k^*) \approx \nu_0 k_d^2. \tag{26}$$

Using (23), (22) and (13) with $\bar{\chi}$ replaced by $\overline{\chi^*}$, this yields

$$\overline{\chi^*}^{4/3} k^{*-2/3}/\nu_0 \sim \nu_0 k_d^2. \tag{27}$$

Eliminating k^* from (25) and (27), we get

$$\overline{\chi^*}(T) \sim \nu_0^{3/2} [\bar{\chi}(T)]^{1/2} k_d. \tag{28}$$

In the range $K < k < k^*$, the inverse cascade continues with the flux $\overline{\chi^*}$ and is stopped at the wavenumber K (when $T = T_c'(R)$), where the inverse timescale matches the strain rate:

$$D(K) K^2 \approx \nu_0 k_d^2. \tag{29}$$

Using (14) with $\bar{\chi}$ replaced by $\overline{\chi^*}$, this yields

$$[\overline{\chi^*}(T_c')]^{1/3} K^{4/3} \approx \nu_0 k_d^2. \tag{30}$$

which, using (28), gives

$$\bar{\chi}(T_c') \sim k_d^{10} \tag{31}$$

which, upon using (16) and (4), finally reduces to

$$\Delta T_c = T_c - T_c'(R) \sim R^{3/2\bar{\nu}}. \tag{32}$$

This gives the value

$$\lambda = 3/2\bar{\nu} = 2.4 \tag{33}$$

for $\bar{\nu} = \frac{5}{8}$.

Thus, as a result of our calculations, we get the value of the exponent λ to be 1.44 and 2.4 from (20) and (32), respectively.

4. Conclusion

In the model proposed, we began with the observation that the microscopic Reynolds number at scales k_m^{-1} is only $\sim 10^{-2}$, which allowed us to neglect any effects of the random stirring in the production mechanism of concentration fluctuations through the Cahn–Hilliard instability. Further, following Ruiz and Nelson's arguments, we considered the active coupling term in (9) to be of importance in the dynamics of small scales. This term supports an inverse cascade in the hydrodynamic regime ($k < k_m$). This term also supports Alfvén-like waves, which converts concentration gradients into velocity fluctutations which undergo subsequent dissipation by viscosity, since all the processes take place in the viscous range ($k > k_d$) in the experimental conditions. Since the Reynolds number R determines the viscous cutoff k_d, we could relate the depression ΔT_c to R without any difficulty. In one extreme limit, when we assumed no dissipation, we obtained $\lambda = 1.44$. In the other extreme limit, when we assumed *all* the dissipation is concentrated at a wavenumber determined by the steady-state condition, we obtained $\lambda = 2.4$. The latter value should be regarded as more close to the realistic situation, because the steady-state condition has been taken into account in obtaining this result. Thus the assumption of *active* participation of concentration gradients in the dynamics of the velocity field leads to reasonably good agreement with the experimental result.

Since we have considered only the above two theoretical extremes, it would be interesting to find a way of calculating the exponent without making such theoretical assumptions for the intermediate case, as well as finding out how the result may depend on any experimental parameter. The only way to do so seems to be by means of a fractal model which takes account of the dissipation of the concentration gradient fluctuations by the Alfvénic equipartition *at every step* of the backward cascade, as well as by properly taking the balance between the production and dissipation processes into account. Such a fractal model would involve the viscosity (an experimental parameter) in the equations, because of the presence of the attenuation factor $(1 - f)$ (shown following equation (23)). As the calculations presented in this work are only two extremes, such a fractal model is expected to yield a number between 1.44 and 2.4, with, presumably, weak dependence on viscosity.

Acknowledgments

The author wishes to thank Professor J K Bhattacharjee of the Department of Physics, Indian Institute of Technology, Kanpur, for various discussions and suggestions, and for careful reading of the manuscript. A part of this work has been supported by the project DST/PHY/9154.

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